# Hamilton Paths for Rectangle Grid Graphs with Triangle Holes

ZHUO (CECILIA) CHEN, Bryn Mawr College, USA

Additional Key Words and Phrases: Hamilton paths, grid graph

**ACM Reference Format:**

Zhuo (Cecilia) Chen. 2025. Hamilton Paths for Rectangle Grid Graphs with Triangle Holes. 1, 1 (November 2025), 11 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

## 1 INTRODUCTION

For a salesman to visit all clients that they need to visit in a given day, with the starting point $s$ as their office and ending point $t$ as their home, the best scenario is moving along the shortest path that can visit every client, which they wish to visit exactly once [7]. This was proved to be NP-complete. This is equivalent to the Hamilton path problem, which is finding a path in an undirected or directed graph that visits each vertex exactly once. This was proved to be NP-Complete [2].

In 1982, Itai et al. have proposed a simpler version of the traveling salesman problem [2]. Instead of considering the shortest tour in Euclidean distance for a general situation, we can consider clients that need to be visited as vertices in a planar grid graph. A planar grid graph is a finite node-induced subgraph of the two-dimensional infinite grid. It is completely specified by vertex set since two vertices are connected if and only if the Euclidean distance between them is equal to 1. Moving in the grid graph can only follow the grid, either horizontally or vertically, which maps the distance calculation into the Manhattan distance. The planar grid graph is a graph on the integer lattice whose vertices are any set of points and whose edges connect every pair of vertices at a unit distance. A planar grid graph is solid if it does not have any holes inside that disconnect its complement in a planar integer lattice. In this case, finding the shortest tour could be simplified into finding the shortest tour in a solid planar grid graph.

This problem is important because the traveling salesman problem (TSP) has far-reaching applications beyond a salesperson's itinerary—it models fundamental optimization challenges in areas such as logistics, circuit board design, vehicle routing, and scheduling. As discussed by Hoffman, Padberg, and Rinaldi, TSP instances frequently arise in practical scenarios where minimizing travel cost or time is essential, and the structure of the problem lends itself to a wide variety of real-world constraints[1, 7]. By focusing on grid-based variations, like those on planar grid graphs, we can develop specialized algorithms that are more efficient for constrained environments such as urban delivery routing or warehouse robotics, where movement is naturally restricted to a grid-like layout.

This paper studies Hamiltonian paths in rectangular grid graphs with triangular holes, a problem that arises in circuit routing and path planning. We propose to cut the input graph into smaller and simpler subgraphs, each of which we can solve through case analysis. Then we will combine the individual Hamiltonian paths in the subgraphs into a

Author's address: Zhuo (Cecilia) Chen, Bryn Mawr College, Bryn mawr, USA, zchen4@brynmawr.edu.


Manuscript submitted to ACM

single Hamiltonian path for the entire graph, if possible, or show that no Hamiltonian path exists. We first establish foundational results for the existence of Hamiltonian paths in rectangle grids with exactly two rows (2-rectangle grids). We show that Hamilton paths in such graphs must adhere to strict turn constraints. These constraints are formalized via parity conditions and vertex-position requirements, with explicit criteria for path existence.

Our primary contribution is a systematic analysis of grids with triangular holes. By decomposing such holes into stacked T-shapes, we derive necessary conditions for Hamiltonian paths, including geometric restrictions on hole placement (e.g., proximity to boundaries) and parity-driven prohibitions. For instance, we prove that no Hamiltonian path exists when the hole's height is odd and its top vertex is adjacent to a grid edge (Theorem 5.1), or when the grid's width is odd and endpoints lie above the hole (Theorem 5.2). These results advance the theoretical understanding of obstacle-avoiding paths in structured environments.

## 2 RELATED WORK

In 1982, Itai et al. has

proved the NP-completeness for general grid graphs' Hamilton path (and circuit) problem in Hamilton Paths in Grid Graphs[2]. They also proposed necessary and sufficient conditions for a rectangle graph to have a Hamilton path between two nodes. A grid graph is rectangular if the size of the node set is the product of two intervals. Given some $m$ and $n$, a grid graph is rectangle if it is isomorphic to $R(m, n)$, whose vertex set $V(R(m, n)) = \{v : 1 \leq v_x \leq m \text{ and } 1 \leq v_y \leq n\}$.

They proposed that if there is a $s, t$ Hamilton path in $G$, $G$ has to be acceptable, which has to be color compatible and not forbidden. For a graph to be color-compatible in black and white, if:

(1) the number of vertices is even, then $s$ and $t$ has to have different colors
(2) the number of vertices is odd, then $s$ and $t$ are colored by the majority color.

For a graph $G$ to be not forbidden, we need $\{s\}$, $\{t\}$, and $\{s, t\}$ cannot be a separating vertex set, i.e. $G - \{s\}$ (the graph $G$ with $s$ removed), $G - \{t\}$, and $G - \{s, t\}$ have to be connected. They have mentioned three different scenarios against this case but is only constrained to if $G$ is a 1-rectangle, 2-rectangle, and specific $3 * 2k$ scenario for some integer $k$.

Furthermore, they proved some sufficient conditions for the Hamilton Path problem to have solutions. Based on those, they proposed a time-linear in the length of the path, $O(mn)$ algorithm to decide whether a Hamilton path problem for $(R(m, n), s, t)$ has a solution.

While Itai et al. established NP-completeness for general grid graphs and provided efficient algorithms for rectangular cases, Umans and Lenhart later investigated a key restriction: solid grid graphs (those without holes) [8]. In their paper "Hamiltonian cycles in solid grid graphs", they showed that the Hamiltonicity problem for such graphs admits a polynomial-time solution, contrasting with the NP-hardness of the general case. Specifically, they proved that deciding whether a solid grid graph contains a Hamiltonian cycle—a closed tour visiting every vertex exactly once—can be done efficiently. This result not only refines the boundary between tractable and intractable instances of the problem but also highlights the critical role of holes in grid graphs' computational complexity. Their work complements Itai et al.'s linear-time algorithm for rectangular graphs by extending efficient decidability to a broader class of hole-free grid structures.

Building upon the foundational work of Itai et al. and Lenhart and Umans, Keshavarz-Kohjerdi and Bagheri investigated Hamiltonian paths in more complex grid graph structures with holes [3–6]. The authors categorized the grid graphs with holes into a number of shapes based on the hole geometry, which they named C-shaped, L-shaped, O-shaped, and so on. The O-shaped grid is a rectangular grid graph with a rectangular hole inside, with four edges of the smaller rectangle not on the board of the larger rectangle. The C-shaped grid is the grid graph with one edge of the rectangle In [3], they explored the sufficient and necessary conditions for a C-shaped grid graph to have a Hamilton path. They also present a linear-time algorithm for finding the Hamilton path. In [4], they proposed the sufficient and necessary conditions, and also a linear-time algorithm for the L-shaped grid graph. In [5, 6] respectively, they focus on so-called O-shaped graphs, which are rectangular grid graphs containing a rectangular hole, where the hole does not share any border sides with the outer rectangle. The authors establish necessary conditions for the existence of Hamiltonian $(s, t)$-paths in such graphs, extending the concept of acceptability to include additional forbidden conditions specific to graphs with holes. Their main contribution is a linear-time algorithm that either finds a Hamiltonian path between given vertices $s$ and $t$ or determines that no such path exists under their necessary conditions.

The algorithm employs strategic graph separations into rectangular, L-shaped, C-shaped, and O-shaped subgraphs, then combines Hamiltonian paths or cycles from these components. This work partially resolves an open problem posed by Lenhart and Umans regarding polynomial-time algorithms for grid graphs with restricted holes, while also having practical applications in areas like robotic path planning with obstacles and maze generation [8].

## 3 PRELIMINARY SUPPORTING THEOREMS

We start by looking at 2-rectangle grid graphs. Recall that 2-rectangle grid graphs are rectangular grid graphs with exactly two rows or two columns. In other words, $R(2, n)$, or $R(m, 2)$. We would also like to specify some start and/or end point of the Hamiltonian path we seek. In other words, we require the Hamiltonian path to be between two vertices, some $s$ and $t$. The ability to specify starting and ending vertices will allow us to use these 2-rectangle grids as building blocks to construct Hamiltonian paths in larger graphs.

We will focus on specific cases where the 2-rectangle grid is $R(m, 2)$, and $s$ and $t$ are in the leftmost or the rightmost column. We show that a Hamilton path exists when certain conditions hold and furthermore, they are also sufficient conditions.

### 3.1 Two Possible Strategies

There are only two possibilities for constructing a $s$-$t$ Hamiltonian path in $R(m, 2)$, where $s$ and $t$ are either in the leftmost column or the rightmost column. In other words, we enter and exit the rectangle from the sides - either we enter and exit on the same side, or we enter on one side and exit from the other. Starting from $s$, because we have a grid graph, we can either travel vertically or horizontally. We will keep going until we run out of vertices and then turn.

Proposition 3.1. *In a 2-rectangle $R(m, 2)$, for a $s$-$t$ path to be Hamiltonian (where $s$ and $t$ are either in the leftmost or the rightmost column), we need to choose one of the following two strategies:*

*(1) keep going vertically and turn when necessary. A turn is necessary when we run out of vertices in the direction we are going. (Fig. 1a);*
*(2) keep going horizontally and turn when necessary. A turn is necessary when we run out of vertices in the direction we are going. (Fig. 1b).*

Note that because $R(m, 2)$ has only two rows, strategy (1) necessarily results in successive U-turns, and strategy (2) results in a single sideways U. Note that $R(2, n)$ is symmetric and we will not discuss it separately.

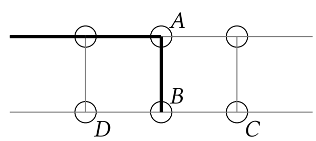
(a) (b)

Fig. 1. $s$ and $t$ are denoted by the solid black vertices

Proof. We will prove Proposition 3.1 by contradiction. Note that both of our strategies require that we turn only when necessary - that is, when we run out of vertices in the direction of travel. In contradiction, the only remaining possibilities are to turn when not necessary or to not turn when necessary, which we show in the following two cases:

(a). **Unnecessary turn while going horizontally**: We were in case (2), and we have been going horizontally, but we make an unnecessary turn before we reach the right-most column, which is shown in Fig. 2. Let vertex $A$ be the first vertex where such a turn happened. Turning at $A$ will lead the path to vertex $B$. Note that both vertices $C$ and $D$ have not been visited at this point, because we have been traveling horizontally in the upper row, without turns.

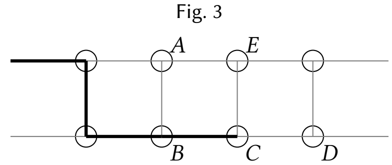
Fig. 2




Since this is a 2-rectangle with height 2, the only ways we can go from $B$ are to visit vertex $C$ or $D$. If we visit $C$, then we can only move towards the right (compared to where $A$ and $B$ are at), $D$ is never reachable. Otherwise, if we visit vertex $D$ instead, the path of moving to the right has been blocked since both $A$ and $B$ have already been visited, and we cannot visit the same vertex more than once. Hence, vertex $C$ can never be visited. Therefore, this cannot be a Hamiltonian path.

(b). **Not turning when going vertically**: We were in case (1) and at vertex $B$, instead of turning to keep going vertically towards $A$, we go horizontally towards $C$, as shown in Fig. 3. Let vertex $B$ be the first vertex where this happened.

Fig. 3




Again, since this is a 2-rectangle with height 2, there are only two possibilities for the next step from $C$: either turn to vertex $E$ or going straight towards vertex $D$.

Case I: if we turn to vertex $E$, then the same difficulty as described in (a) will happen: either vertex $A$ cannot be visited, or vertices to the right of $E$ cannot be visted.

Case II: otherwise, if we go straight towards $D$, we can only choose to keep going straight until we reach the rightmost vertex. Note that if at any point we turn towards the top row the same contradiction as described in case (a) will again occur. Then, if we keep going straight, when we reach the rightmost vertex, we will still have many vertices in the top row unvisited, starting from $A$. If $t$ is in the right column then we must exit and can not cover the top row. If $t$ is in the left column and we turn at the rightmost column to go back, we will be stuck at vertex $A$ and can not reach $t$. Either way, the path can not be Hamiltonian.

Therefore, to turn when not necessary and to not turn when necessary are both invalid strategies for constructing a Hamiltonian path. □

### 3.2 Two strategies as the Necessary Conditions

We will now show that the two strategies are not interchangeable when given specific combinations of $s = (s_x, s_y)$ and $t = (t_x, t_y)$ positions. Recall that $s$ and $t$ must be either in the leftmost column or the rightmost column. For the rest of the discussion of $R(m, 2)$ in this section, the requirement that $s$ and $t$ are either in the leftmost column or the rightmost column will remain, and therefore when we say $s$ and $t$ are in the same column, we really mean that $s$ and $t$ are both in the leftmost or the rightmost, or when we say that $s$ and $t$ are not in the same column, we mean that one is in the leftmost and the other is in the rightmost. Again, symmetrical cases will not be discussed.

Lemma 3.2. *For a 2-rectangle grid graph $R(m, 2)$ to have a valid Hamilton path:*

*(1) $s$ and $t$ are in the same column. Then $s_x = t_x$. A Hamiltonian path exists between $s$ and $t$ and we can find it by following strategy 2 in Proposition 3.1 - to go horizontally and complete a single sideways U, as shown in Figure 4a.*
*(2) $s$ and $t$ are not in the same column. Then $s_x \neq t_x$, and we need to follow strategy 1 in Proposition 3.1 - to go vertically and perform successive U-turns, as shown in Figure 4b;*

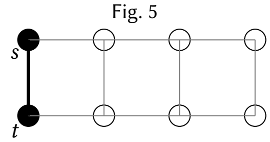
(a) (b)

Fig. 4. $s$ and $t$ are denoted by the solid black vertices

Proof. We analyze the cases by contradiction.

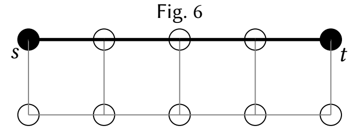
(1) When $s_x = t_x$, if instead of strategy 2, we follow strategy 1 in Proposition 3.1. As shown in Figure 5, starting from vertex $s$, the path will just go vertically, directly linking $s$ and $t$. Along this path, all other vertices besides $s$ and $t$ are not visited. Therefore, the only possible situation for $R(m, 2)$ with $s_x = t_x$ to have a valid Hamilton path is to go horizontally, since there are only two valid strategies based on Proposition 3.1. ■

Fig. 5



(2) When $s_x \neq t_x$, if instead of following strategy 1, we follow strategy 2 in Proposition 3.1. We then have the situation that the path connecting $s$ and $t$ is a horizontal line (or a horizontal line followed by a small turn if $t_y \neq s_y$). As shown in Figure 6, all vertices with a different $y$ value than $s$ will not be covered at all since the path already ended at $t$. ■

Fig. 6




□

Note that when $s$ and $t$ are not in the same column, even if we use strategy 1, a Hamiltonian path still may not exist. In order for strategy 1 to cover all vertices in both rows, we need to keep doing U-turns, or a vertex will be left uncovered. Each U-turn needs 2 columns and therefore the parity of the number of columns also needs to be considered, in addition to the $s$ and $t$ positions. In particular, because each successive U-turn goes in the opposite direction, if $s$ and $t$ are in the same row, we need to compose an odd number of U-turns which requires an even number of columns. Similarly if $s$ and $t$ are not in the same row, we need to compose an even number of U-turns, which requires an odd number of columns.

Proposition 3.3. *The 2-rectangle grid graph $R(m, 2)$ does not have a valid Hamilton path between $s$ and $t$, if $s$ and $t$ are not in the same column and*

*(1) $s$ and $t$ are not in the same row and $m$ is even.*
*(2) $s$ and $t$ are in the same row and $m$ is odd.*


Fig. 7

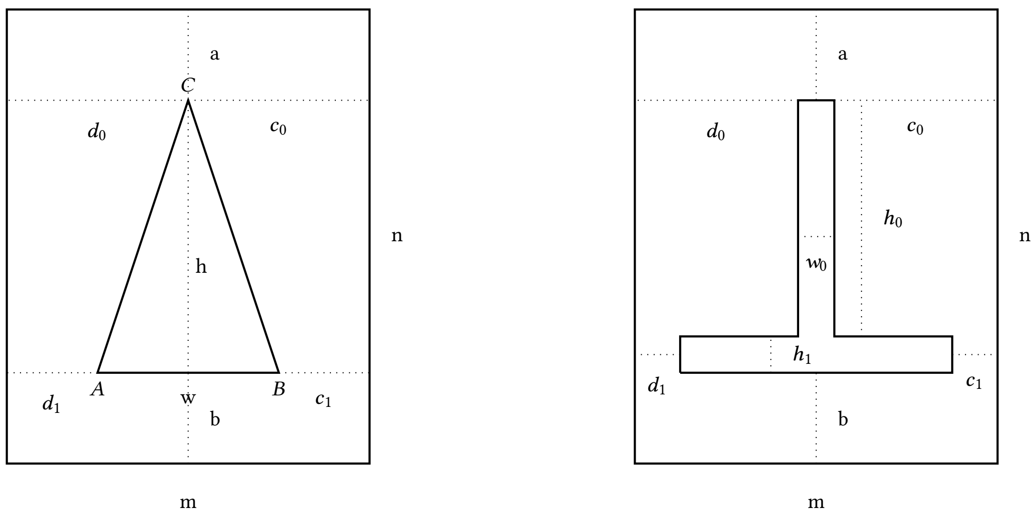
Proof. Given that $s$ and $t$ are vertices in the leftmost or rightmost column but not both in the same column, we know $s_x \neq t_x$. Then, by Lemma 3.2, we know that the only possible Hamilton path is by going vertically and performing U-turns.

(1) Because $s$ and $t$ are not in the same row, we need an even number of U-turns which requires $m$ to be odd. If $m$ is even, the vertex above $t$ in the rightmost column will be uncovered as shown in Figure 7a.
(2) Because $s$ and $t$ are in the same row, we need an odd number of U-turns which requires $m$ to be even. If $m$ is odd, the vertex below $t$ in the rightmost column will be uncovered as shown in Figure 7b.

□

We hope to approach the Hamiltonian path problem in more complex grid graphs by finding a way to separate it into 2-rectangle subgraphs with specific entry and exit vertices. We can then construct $s-t$ Hamilton paths in these 2-rectangle grids with existing results and then connect them together.

## 4 SETUP OF THE TRIANGLE HOLE




(a) A triangle hole inside of a rectangle grid graph

(b) Convert the thin triangle hole into a T shape hole

Fig. 8

We will now explore the necessary conditions for a rectangle grid graph with a rectangle hole to have a Hamilton path, shown in Figure 8. We only focus on the situation where the triangle is placed with at least one edge parallel to an edge of the outer rectangle grid graph. For example, in Figure 8a for triangle $ABC$, edge $AB$ is parallel to the bottom edge of the outside rectangle.

In a rectangle grid graph, $R(m, n)$, where, WLOG, $m$ is the width and $n$ is the length, we define a triangle hole $T(w, h)$, where $w$ is the base of the triangle (the edge that is parallel to the bottom edge of the outer rectangle), and $h$ is the height. We set the following variables: $a$ as the distance between the upper edge of the rectangle and the apex of the triangle; $b$ as the distance between the triangle's base and the rectangle's bottom edge; $d_0$ and $c_0$ are the distances between the triangle's apex and the left and right edges of the rectangle respectively; and $d_1$ and $c_1$ are the distances between the end points of the base and the left and right edges of the rectangle respectively. Note that triangle $T(w, h)$ is not necessarily centered in the rectangle grid graph.

### 4.1 The Thin Triangle

Given a rectangle grid graph $R(m, n)$ with a triangle hole $T(w, h)$, if we add the requirement that: each layer of the triangle has width 1, except for the bottom layer, which has arbitrary width, as long as it fits inside the rectangle. We call the triangle represented by a grid graph holding the above property a **thin triangle**. A thin triangle is equivalent to a reversed T-shape, shown in Figure 8b, where the height for the horizontal part, $h_1 = 1$, and the width for the vertical part, $w_0 = 1$.

### 4.2 Thin Triangles Stacking

Now, we can stack several reversed T-shape holes together, by arranging the bottom rows layer by layer from the widest to the thinnest, as shown in Figure 9. In addition, we require that all T-shape holes have the vertical column aligned at the same $x$ value. This is equivalent to all T-shape holes having the same $d_0$, $c_0$, and $w_0$ value. Note that since the T-shapes are stacked from the widest to the thinnest, the spaces between the T-shapes stack and left/right edges decrease gradually from top to bottom, i.e, $d_0 \geq d_1 \geq d_2 \geq \cdots \geq d_n$ and $c_0 \geq c_1 \geq c_2 \geq \cdots \geq c_n$.

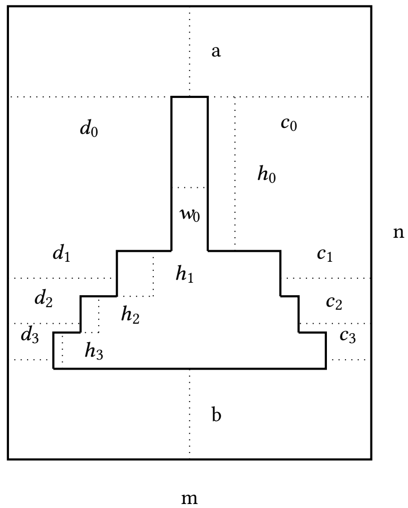



Fig. 9. Convert several thin triangle holes into a triangle hole

It is not difficult to see that an arbitrary triangle can be obtained by stacking a specific sequence of thin triangles, where each T's base width is equal to the base width of the similar triangle with the same height

Given this construction, we define $w_0$ as the width of the middle column, which is the same as any other $w_0$ value for all T-shapes. We have $h_0$, which is the height of the vertical column and also the height of the topmost T-shape. We have $a$, $b$, $d_0$ and $c_0$, defined similarly as in the thin triangle case. The set of $D = \{d_1, d_2, \cdots, d_n\}$ and $C = \{c_1, c_2, \cdots, c_n\}$ are defined by the distance between the base and the leftmost/rightmost edge for each T-shape.

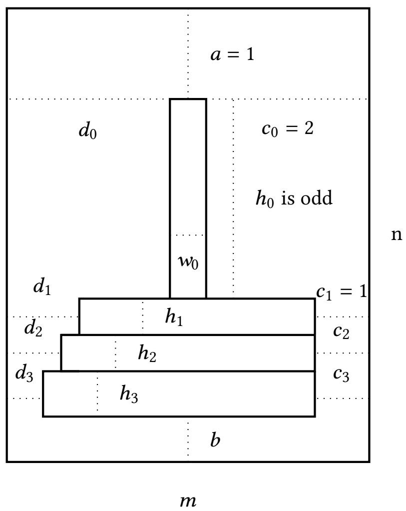
## 5 NECESSARY CONDITIONS

We can now explore the necessary conditions for a rectangle grid graph with a triangle hole. In order to have a Hamiltonian path, every vertex in the grid graph must be visited. Below we examine several cases that do not admit a Hamiltonian path to understand necessary conditions. The key insight is when we separate the grid graph into several subgraphs, if the number of possible starting and ending vertices for each part is sufficiently small, we can reduce what would otherwise be a difficult shape to a combination of more analytically tractable shapes, i.e., several rectangles.

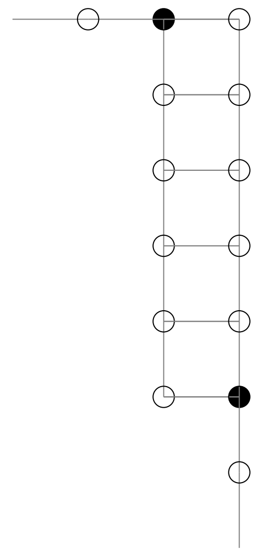
First, we construct the graph so that there is only one possible path on one side of the triangle hole, WLOG, say the right side. Such a rectangle grid graph with a triangle hole will be built by setting $c_1 = 1$. Thus, we also have $c_2, c_3, \ldots, c_n = 1$ since $c_1 \geq c_2 \geq c_3 \geq \cdots c_n$, as shown in Figure 10a. The width 1 vertical path that is necessary to connect the $c_1 \cdots c_n$ area with the rest of the grid graph has unique entrance and exit vertices, which any paths connecting the subgraphs above or below must reach.

Theorem 5.1. *T1. A rectangle grid graph with a triangle hole with $a = 1$ and $h_0$ is odd, does not have a Hamilton path between $s$ and $t$ if any of the following cases hold*

*(1) $c_0 = 2$, $c_1 = 1$, and $s$ and $t$ are outside of the area right-above the triangle hole;*
*(2) $d_0 = 2$, $d_1 = 1$, and $s$ and $t$ are outside of the area left-above the triangle hole.*




(a)



(b)

Fig. 10. $a = 1$, $h_0$ is odd, $c_0 = 2$, and $c_2 = 1$

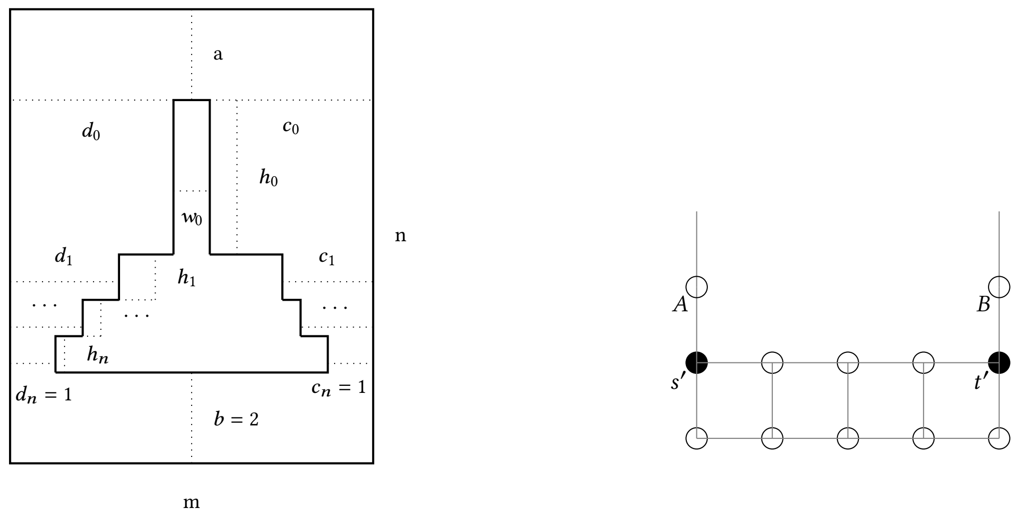
Proof. Let's first discuss the first situation, shown in Figure 10a. Because $s$ and $t$ are outside of the area right-above the hole, we need to connect the right-above area to the rest. Given that $a = 1$ and $c_1 = 1$ (and so that $c_1, \cdots, c_n = 1$), there are only two possible connecting grids that connect the right-above part to the rest, two solid dots shown in Figure 10b. Hence, in order to have a valid Hamilton path in this grid graph, containing a Hamilton path in the upper right part with starting and ending vertices as those two connecting vertices is the necessary condition. Otherwise, if even the upper right part does not have a Hamilton path, i.e., no path that visits any vertex inside the upper right 2-rectangle, then there cannot be a path that visits any vertex for the whole grid graph containing all vertices in the upper right part.

Hence, let's show that the upper right 2-rectangle with those two points as starting and ending points does not have a Hamilton path. If we rotate this 90 degrees counterclockwise, we will have a rectangle $R(m', 2)$, where, WLOG, $s = (0, 0)$ and $t = (m' - 1, 1)$. Given $a = 1$, $m' = a + m = 1 + \text{ odd } = \text{ even }$. Since $s_y \neq t_y$, also $m'$ is even, followed by case 1 in Proposition 3.3, we conclude that this upper right 2-rectangle does not have a Hamilton path. Thus, there is no way the whole grid graph has a Hamilton path.

Similarly, the second situation can be proved in the same way. Since $d_0 = 2$, $d_n = 1$, and $s$ and $t$ are outside of this upper left part, then we need the upper left part to have a Hamilton path, since those two are the only connecting vertices to the other part of this grid graph. We can transform this into a 2-rectangle $R(m'', 2)$ with $s_y \neq t_y$ and $m''$ is even. Thus, similarly by case 1 in Proposition 3.3, we know that this cause the upper left doesn't have a legit Hamilton path, which cause the Hamilton path for the whole grid graph impossible to exist.

□

Theorem 5.2. *T2. A rectangle grid graph with a triangle hole with $b = 2$, $c_n = d_n = 1$, and $m$ is odd, does not have a Hamilton path if $s_y > 2$ and $t_y > 2$.*




(a) (b)

Fig. 11. $b = 2$, $m$ is odd, $c_1 = 1$, and $d_1 = 1$

Proof. As shown in Figure 11a, when $d_n = c_n = 1$, $b = 2$ and if , both $s$ and $t$ are not below the thin triangle, i.e. $s_y > 2$ and $t_y > 2$. In order to have a Hamilton path, we need to connect the vertices above the bottom row of the triangle hole (above $y = 2$) and the vertices below it (below $y = 2$).

Since $d_n = c_b = 1$, and $s$ and $t$ are both above it, we have to enter the bottom rectangle part via the vertex on the left of the triangle's bottom part, and exit on the right (or enter on the right and exit on the left), which are $s'$ and $t'$ vertices labeled in Figure 11b. This is because those two connect to $A$ and $B$, which are the only two possible vertices connecting the lower part and the upper part, and the Hamilton path cannot revisit the same vertex twice. Therefore, WLOG, let's say we have to enter the bottom part via $s'$ and exit via $t'$. This means we can treat the vertices $s'$ and $t'$ as the starting and ending vertices for the lower part.

Hence, we have converted the whole situation into finding the Hamilton path of the 2-rectangle grid graph $R(m, 2)$ between $s'$ and $t'$. If there doesn't exist a Hamilton path for this 2-rectangle, then there for sure does not exist a Hamilton path for the whole rectangle grid graph with a triangle hole. We know that $s'_x = 1$ and $t'_x = m$ and $s'_y = t'_y$, by Proposition 3.3, since $m$ is odd, $R(m, 2)$ does not have a valid Hamilton path between $s'$ and $t'$. Hence, the whole grid graph does not have a valid Hamilton path. □

## 6 CONCLUSION

In this paper, we have proved some necessary and sufficient conditions for the 2-rectangle grid graph to have a Hamilton path by proving the only two possible moving strategies for 2-rectangles. Then, by using the 2-rectangle's sufficient and necessary conditions, we have explored the necessary conditions for a rectangle grid graph with a triangle hole to have a Hamilton path. The future work will include exploring other sufficient/necessary conditions, and even with a more complex triangle hole.

## 7 LITERATURE REVIEW

## REFERENCES

[1] Karla L Hoffman, Manfred Padberg, Giovanni Rinaldi, et al. 2013. Traveling salesman problem. *Encyclopedia of operations research and management science* 1 (2013), 1573–1578.

[2] Alon Itai, Christos Papadimitriou, and Jayme Szwarcfiter. 1982. Hamilton Paths in Grid Graphs. *SIAM J. Comput.* 11 (11 1982), 676–686. https://doi.org/10.1137/0211056

[3] Fatemeh Keshavarz-Kohjerdi and Alireza Bagheri. 2016. Hamiltonian Paths in C-shaped Grid Graphs. arXiv:1602.07407 [cs.CC] https://arxiv.org/abs/1602.07407

[4] Fatemeh Keshavarz-Kohjerdi and Alireza Bagheri. 2016. Hamiltonian paths in L-shaped grid graphs. *Theoretical Computer Science* 621 (2016), 37–56. https://doi.org/10.1016/j.tcs.2016.01.024

[5] Fatemeh Keshavarz-Kohjerdi and Alireza Bagheri. 2017. A linear-time algorithm for finding Hamiltonian (s, t)-paths in odd-sized rectangular grid graphs with a rectangular hole. *J. Supercomput.* 73, 9 (Sept. 2017), 3821–3860. https://doi.org/10.1007/s11227-017-1984-z

[6] Fatemeh Keshavarz-Kohjerdi and Alireza Bagheri. 2017. A linear-time algorithm for finding Hamiltonian (s,t)-paths in even-sized rectangular grid graphs with a rectangular hole. *Theoretical Computer Science* 690 (2017), 26–58. https://doi.org/10.1016/j.tcs.2017.05.031

[7] Rajesh Matai, Surya Prakash Singh, and Murari Lal Mittal. 2010. Traveling salesman problem: an overview of applications, formulations, and solution approaches. *Traveling salesman problem, theory and applications* 1, 1 (2010), 1–25.

[8] C. Umans and W. Lenhart. 1997. Hamiltonian cycles in solid grid graphs. In *Proceedings 38th Annual Symposium on Foundations of Computer Science.* 496–505. https://doi.org/10.1109/SFCS.1997.646138